

disk, streaming tape storage, optical storage, and a wide-bandwidth Megascan monitor which displays the images at full resolution.

Engineering measurements have been made to determine the basic characteristics of the system. The horizontal resolution was measured to be 3.5 line-pairs/mm (lp/mm) with the vertical resolution being 1.6 lp/mm. The minimum signal contrast which can be detected in a large-area feature is 0.2%. Dose to the patient for a standard lateral projection chest image was measured to be 3.3 mR, which is significantly less than that for a typical film/screen chest image.

A number of image postprocessing techniques have been examined for further improving the image quality of the DADR system. Unsharp masking, local contrast stretching, and modified local contrast stretching have all been used. The modified local contrast stretching has produced the best results to date, with image quality exceeding that of standard film/screen chest images. Images have been postprocessed using facilities of the Pittsburgh Supercomputer Center.

Future work will include further refinement of system hardware and software and the pursuit of improved image postprocessing enhancements. Of particular interest are modified local contrast stretching and adaptive techniques such as those which have been suggested in the literature [9], [10]. Scantech Corporation has been pursuing commercialization of DADR medical imaging as well as modification of this technology to produce commercial industrial NDT products, concentrating in the inspection of electronic assemblies and other small parts. Further research is also being considered in the area of dual-energy techniques for chest imaging and bone density measurements through dual energy absorptiometry.

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Comments on "A Cone-Beam Filtered Backprojection Reconstruction Algorithm for Cardiac Single Photon Emission Computed Tomography"

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Abstract—In the paper by Gullberg and Zeng [1], a fan-beam reconstruction formula of a noncircular scanning locus was derived and extended for half-scan cone-beam reconstruction. However, their reconstruction formula is not exact mathematically unless a necessary condition is satisfied. In this correspondence, we will derive this necessary condition and provide a geometrical explanation of the condition.

In a recent paper by Gullberg and Zeng [1], an extended filtered-and-backprojection fan-beam reconstruction formula was derived where the source-to-origin distance is angular dependent. This formula was then extended to cone-beam geometry for half-scan reconstruction where only projection data from a less-than-360° angular span were used. In their derivation of the extended fan-beam reconstruction formula, the parallel-beam reconstruction formula was used as the starting point. Then, a Jacobian transform was performed to change the variables of parallel-beam projection data to those of fan-beam projection data. The extended fan-beam reconstruction formula was obtained directly after the Jacobian transform.

In [1], the absolute value operation was performed on the Jacobian [1, eq. (12)], which suggested that the sign of the Jacobian was allowed to change. However, the change of variables is valid only under the condition that the mapping between the parallel-beam variables and fan-beam variables is one-to-one [2, pp. 768-769]. Based on the theorem on inverse mapping [2, pp. 701-702], if the Jacobian is always greater than zero (or always less than zero), the mapping is one-to-one. The Jacobian $J(\alpha, \xi) = (Dd(\alpha) - d_\alpha(\alpha)\xi)Z^3D$ must be positive, because $J(\alpha, 0)$ is positive. Therefore, the following condition must be satisfied:

$$Dd(\alpha) - d_\alpha(\alpha)\xi > 0. \quad (1)$$

If $-\xi_{\max} \leq \xi \leq \xi_{\max}$, then the condition becomes

$$|d_\alpha(\alpha)| < \frac{Dd(\alpha)}{\xi_{\max}}.$$

The condition (1) can be explained geometrically as follows. In Fig. 1, $d(\alpha)$ denotes the distance between the source and the origin, α is the rotation angle, $d_\alpha(\alpha)$ is the derivative of $d(\alpha)$ with respect to α . Geometrically, \overline{AC} (the distance between the source and the detector array) is equal to a constant D . $\overline{AO} = d(\alpha)$, and $\overline{A'B} = d_\alpha(\alpha)d\alpha$ if $d_\alpha(\alpha) > 0$. Because $d\alpha = ds/d(\alpha)$, where ds is the arc differential,

$$\tan \angle BAA' = \frac{d_\alpha(\alpha)}{d(\alpha)}. \quad (2)$$

As $\angle OAB = 90^\circ$,

$$\tan \angle BAA' = \frac{1}{\tan \angle C'AC}. \quad (3)$$

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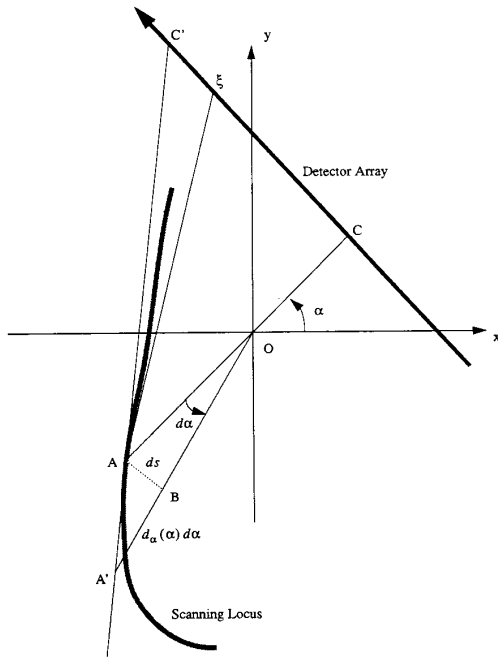


Fig. 1. A geometrical description of the mathematical condition necessary to apply the fan-beam reconstruction formula given in [1]. The direction tangent to the scanning locus must stay outside of the fan delimited by the object support.

Combining (2) and (3), we have

$$\tan \angle C'AC = \frac{d(\alpha)}{d_\alpha(\alpha)}. \tag{4}$$

On the other hand,

$$\tan \angle \xi AC = \frac{\xi}{D}. \tag{5}$$

Substituting (4) and (5) into (1), we have

$$\tan \angle C'AC > \tan \angle \xi AC$$

or

$$\angle C'AC > \angle \xi AC. \tag{6}$$

Evidently, (6) means that the direction tangent to the scanning locus must stay outside of the fan delimited by the object support. If $d_\alpha(\alpha) \leq 0$, the same geometrical explanation can be similarly obtained. In fact, if the condition (1) is satisfied, any straight line passing through a point in the object support must only intersect the scanning locus twice; otherwise, some lines will intersect the locus more than twice.

Generally speaking, if the sign of the Jacobian changes, the mapping is not one-to-one, and thus the fan-beam reconstruction formula given in [1] is no longer exact. In fact, the one-to-one requirement for the Jacobian transform has been well noticed in the previous literature, such as [3], where the Jacobian transform was used on appropriate subintervals so that the change in variables can be performed. Similar comments can be applied to another paper [4] published in the same issue, where the Jacobian [4, eq. (9)] is implicitly assumed to be positive without any discussions.

Since a good experimental setup would not vary the scanning locus too drastically, the Jacobian for a practical scanning locus, e.g., an elliptical or polygonal trajectory, is always positive. As a result, the calculations of [1] remain valid. Therefore, this correspondence is mainly a mathematical remark. It will be interesting to consider a fan-beam reconstruction formula in the case where the mapping is not one-to-one. The redundant projection data should be utilized in such a way that the reconstruction error is minimized.

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