Theoretical FWTM values in helical CT

G. Wang, J. A. Brink, and M. W. Vannier
Mallinckrodt Institute of Radiology, Washington University School of Medicine, Saint Louis, Missouri 63110

(Received 24 February 1994; accepted for publication 23 March 1994)

Helical computed tomography (helical/spiral CT) permits rapid scanning and has been clinically accepted. In our recent study,[1,2] longitudinal image resolution in conventional and helical CT was analytically characterized in terms of the bandwidth of the transfer function (TF) for pitch equal to 1. A bandwidth comparison showed that although the continuous table motion required in helical CT introduces blurring, helical CT actually allows substantially better longitudinal resolution for a given dose due to its inherent retrospective reconstruction capability. In practice, longitudinal resolution is generally described as the full width at one-tenth of the maximum (FWTM) of the section sensitivity profile (SSP). Currently pitch values other than 1 are also in use. Therefore, we tabulate the theoretical FWTM values in helical CT as a function of pitch and reconstructed slice number per collimation length, and discuss the implications of the data.

Let us model a longitudinal detector respond function as $f(z)=(1/D)\mathrm{rect}(z/D)$, where z denotes the longitudinal coordinate, D the longitudinal dimension of the detector collimation, and rect( ) the rectangular function

$$\mathrm{rect}(z)=\begin{cases}1, & z\in[-1/2,1/2),\\ 0, & \text{otherwise.}\end{cases} \tag{1}$$

Assume that pD represents the table increment, the SSP in conventional CT is expressed as the convolution of the detector response function and the low-pass filter function $(1/pD)\mathrm{sinc}(\pi z/pD)$ (ignoring the aliasing effect),

$$p_{c,p}(z)=\frac{1}{pD^2}\,\mathrm{sinc}\left(\frac{\pi z}{pD}\right)*\mathrm{rect}\left(\frac{z}{D}\right), \tag{2}$$

where p is referred to as the pitch (table increment to detector collimation), $\mathrm{sinc}(z)=\sin z/z$. Due to the continuous table motion and resulting interpolation process, the SSP in helical CT is degraded compared to $f(z)$ even if transaxial reconstruction interval is made sufficiently small. If the half-scan interpolation (HI) method is employed,[3,4] the associated table motion function is given as follows:

$$g(z,p)=\begin{cases}\dfrac{2}{pD}+\dfrac{4z}{p^2D^2}, & z\in[-pD/2,0);\\[2mm] \dfrac{2}{pD}-\dfrac{4z}{p^2D^2}, & z\in[0,pD/2);\\[2mm] 0, & \text{otherwise.}\end{cases} \tag{3}$$

Assuming that n transaxial slices are reconstructed per collimation length, the SSP in helical CT is formulated as

$$p_{h,p,n}(z)=\frac{n}{D}\,\mathrm{sinc}\left(\frac{n\pi z}{D}\right)*g(z,p)*f(z). \tag{4}$$

Based on Eq. (4), full width at one-tenth of the maximum (FWTM) values are tabulated in Table I as a function of pitch (p) and reconstructed slice number (n) per collimation length. In the computation, D was set to 5. For each combination of p and n, the functions sinc, g, and f were generated in one-dimensional arrays of 512 elements and $10D$ in length. The convolutions were implemented via fast Fourier transform. The z coordinates corresponding to the one-tenth of the SSP maximum were found via an automatic search process.

According to Table I, the less p is, the more slices should be reconstructed to make full use of helical scan data. Generally, it is recommended that 3 to 5 slices be reconstructed for $p\leq 1$, and that 2 slices reconstructed for $p>1$. Experimental FWTM values in helical CT for $D=5$, $p=1$, and 2 are 8.0 and 11.3, respectively.[4] They are consistent with our theoretical values, which are 7.7 and 11.2, respectively. The tendency of slight under-estimation mainly arises from the idealized shape of the rect function, which results in less blurring in the theoretical SSP. Note that as n increases, the SSP will be broadened, and its maximum decreased. The FWTM as a function of n does not always decrease monotonically. It can be observed that the minimum FWTM for $p=2$ was reached at $n=2$, which reflects the fact that the FWTM is only an approximate descriptor of the longitudinal image resolution.

Strictly speaking, the real system response to an impulse (a tiny ball) is neither stationary along the z axis due to the aliasing effect, nor transaxially invariant with respect to the x-y position due to the asymmetric geometry in the helical CT interpolation. The rationale of our SSP formulation is as follows. When a finite sampling is performed, it is usually assumed that the sampling rate is sufficiently high so that the spectrum aliasing is not significant. With the Shannon kernel in our SSPs a proper cutoff frequency of an object (patient) function is implied, since in reality the Shannon interpolation is performed on the discretized convolution of the rect function and the object function instead of the discretized rect function only. In this sense, no aliasing effect is involved, and the SSPs (2) and (4) are correct that describe the system longitudinal response to a low-pass filtered impulse. Similarly, the meaning of our SSP bandwidths is that an original longitudinal signal can be exactly recovered from its discretized version if its maximum frequency stays within the system bandwidth.[1] If the aliasing effect is taken into account in the bandwidth determination, the original physical meaning of the bandwidth will be lost. On the other hand, if a sampling rate is less than the Nyquist rate associated with an object function (a tiny ball and a large reconstruction interval), the aliasing effect will cause errors in the Shannon interpolation. In this case, our SSPs are approximations to nonstationary profiles (the larger n, the higher the accuracy), providing a reasonable basis for performance evaluation. As far as the transaxial variation is concerned, it was demon-

 0094-2405/94/21(6)/753/2/$1.20

TABLE I. Full width at one-tenth of the maximum (FWTM) values as a function of pitch (p, the first column) and reconstructed slice number (n, the first row) per collimation length.

$p\backslash n$	1	2	3	4	5
0.33	9.9	6.9	6.5	6.2	6.0
0.67	10.1	7.5	6.9	6.9	6.7
1.0	10.3	7.9	7.7	7.7	7.7
1.33	10.6	8.7	8.7	8.7	8.7
1.67	11.0	9.9	9.9	9.9	9.9
2.0	11.8	11.0	11.2	11.2	11.2

strated in our study[5] that in terms of the standard deviation-the SSP transaxial variation in helical CT is within about 5% and 10% for fan angles of 40° and 50°, respectively. Therefore, the SSP (4) should be representative in most clinical applications.

ACKNOWLEDGMENT

This work was supported in part by the Whitaker Foundation (Biomedical Engineering Program).

[1]G. Wang and M. W. Vannier, "Longitudinal resolution in volumetric x-ray CT—Analytical comparison between conventional and helical CT," Med. Phys. **21**, 429–433 (1994).

[2]J. A. Brink, J. P. Heiken, G. Wang, K. W. McEnery, F. J. Schlueter, and M. W. Vannier, Spiral (helical) CT: Principles and technical considerations, to appear in the July 1994 issue of RadioGraphics (1994).

[3]C. R. Crawford and K. F. King, "Computed tomography scanning with simultaneous patient translation," Med. Phys. **17**, 967–982 (1990).

[4]A. Polacin, W. A. Kalender, and G. Marchal, "Evaluation of section sensitivity profiles and image noise in spiral CT," Radiology **185**, 29–35 (1992).

[5]G. Wang and M. W. Vannier, "Spatial variation of section sensitivity profile in helical CT," submitted for publication.