Zoological Studies 34, Supplement I (1995)

Feldkamp-type Cone-beam Reconstruction: Revisited

Ge Wang[2] and Ping-Chin Cheng[1]

[1]*Advanced Microscopy and Imaging Laboratory (AMIL), Department of Electrical and Computer Engineering State University of New York, Buffalo, NY 14260, USA*

[2]*Mallinckrodt Institute of Radiology, Washington University, School of Medicine, Saint Louis, Missouri 63110, USA*

The cone-beam approach is desirable for faster data collection, higher image resolution, better radiation utilization and easier hardware implementation, therefore it attracts more and more attention in biological, medical and material studies (Smith 1990, Gulberg 1992, Cheng et al. 1993). Despite important progress in exact cone-beam reconstruction (Tuy 1983, Smith 1985, Grangeat 1991, Danielsson 1992, Axelsson and Danielsson 1994), approximate cone-beam formulas remain practically important. Feldkamp-type formulas are popular in approximate cone-beam reconstruction (Feldkamp et al. 1984, Gullberg 1992, Wang et al. 1991, 1992, 1993a, 1993b, 1994).

The advantages of approximate cone-beam reconstruction are as follows. First, incomplete scanning loci can be used. The completeness condition for exact reconstruction rquires that there exist at least a source position on any plane intersecting an object. This condition cannot be satisfied in cone-beam X-ray microtomography when planar or dashed-line helical scanning loci are used (Wang et al. 1991, 1939a). Second, partial detection coverage is permissible. In exact cone-beam reconstruction, the cone-beam is assumed to cover the entire object from any source position. Unlike emission tomography, complete detection coverage is impossible in cone-beam X-ray microtomography, since most specimens are either rod-shaped or planar instead of spheric. Third, computational efficiency is high. Because of the second advantage, approximate reconstruction involves much less raw data, especially in reconstruction of rod-shaped and planar specimens. The computational structure of Feldkamp-type reconstruction is straightforward, highly parallel, and hardware supported. Feldkamp-type formulas are particularly fast in reconstructing a limited number of slices or small regions of interest. The linogram idea (Edholm and Herman 1987) used in exact fourier cone-beam reconstruction (Danielsson 1992, Axelsson and Danielsson 1994) may also be adapted to Feldkamp-type reconstruction. Fourth, image noise and ringing artifacts are less. With the direct Fourier method (Axellson and Danielsson 1994), it was found that exact cone-beam reconstruction produces more ringing as compared to the Feldkamp method. We hypothesize that this is inherent to all exact cone-beam reconstruction formulas that take the second derivative of data. In most of the cone-beam literature synthetic noise-free data are used, therefore this problem did not look too serious. Further evaluation and comparison would be valuable

Feldkamp-type cone-beam formulas were derived by modifying the convolution and back projection fan-beam formula. To reconstruct a voxel, fan-beams tilted horizontally and passing through the voxel are used. To Compensate for the tilted fan-beam geometry, both the source-to-origin distance and the angular differential were modified, and incremental contributions integrated. In this paper we will formulate Feldkamp-type reconstruction in a manner that is clearer in terms of plausibility.

Let the $\vec{x}$ coordinate system be the cone-beam reconstruction system, where $\vec{x}=(x, y, z)^t$, $(.)^t$ denotes the transpose of a vector. A specimen $s(\vec{x})$ is supported in the cylindrical region $x^2+y^2 \leq 1$. A scanning locus is described as $\vec{\phi}(\beta)=(\rho(\beta)\cos\beta,\ \rho(\beta)\sin\beta,\ h(\beta))^t$, $\rho(\beta)>1$, where β is the X-ray source rotation angle around the z axis counterclockwise. A scanning turn is obtained by restricting β in $[0, 2\pi)$. Cone-beam projection data, $R(p, \zeta, \beta)$, or $R(\vec{\alpha}, \beta)$, are recorded on an imaginary detector plane passing through the z axis and facing the X-ray source, where p and ζ are horizontal and longitudinal coordinates of the detector system, and $\vec{\alpha}=(\alpha_{11}, \alpha_2, \alpha_3)^t$ specifies the direction of an X-ray.

Let us consider δ and *c* functions: $\delta(\vec{x}-\vec{x}_0)$ models a 3-D point object located at $\vec{x}_0$, $c(\vec{x})$ is defined as being independent of z, that is, $c(\vec{x})=c(x, y)$.

Longitudinally projecting $\vec{\phi}(\beta)$, $\beta\in[0, 2\pi)$, onto the $z=z_0$ plane. We have another scanning locus: $\vec{\phi}^*(\beta)=(\rho(\beta)\cos\beta\ \rho(\beta)\sin\beta\ z_0)^t$ $\beta\in[0, 2\pi)$. It can be shown that exact fan-beam data $R^*(p, 0, \beta)$ in the $z=z_0$ plane can be obtained by multiplying a horizontal profile of cone-beam projection $R(p, \zeta, \beta)$ with the cosine of the X-ray tilting angle, $\frac{\sqrt{\rho^2(\beta)+p^2}}{\sqrt{\rho^2(\beta)+p^2+\zeta^2}}$. Note that in the case of $\delta(\vec{x}-\vec{x}_0)$ ζ should be the longitudinal coordinate of projected $\delta(\vec{x}-\vec{x}_0)$.

Under the moderate conditions we proved a derivative-free noncircular fan-beam reconstruction formula (Wang et al. 1993a):

$$g(x,y)=\frac{1}{2}\int_0^{2\pi}\frac{\rho^2(\beta)}{(\rho(\beta)-s)^2}\int_{-\infty}^{\infty}F(p,\beta)f\left(\frac{\rho(\beta)t}{\rho(\beta)-s}-P\right)\frac{\rho(\beta)}{\sqrt{\rho^2(\beta)+\rho^2}}dpd\beta, \tag{1}$$

where $F(p, \beta)$ represents fan-beam data,

$t=x\cos\beta+y\sin\beta$ and $s=-x\sin\beta+y\cos\beta$,

f is a reconstruction filter. Hence, applying the derivative-free noncircular fan-beam reconstruction formula with $R^*(p, 0, \beta)$ will produce exact reconstruction o the $z=z_0$ plane.

As δ and c functions represent typical cases of sharp and smooth variation, it is reasonable to apply the same cone-beam data correction scheme generally to approximate transaxial fan-beam data. By doing so, we immediately obtain the generalized Feldkamp cone-beam reconstruction formula (Wang et al. 1993b):

$$g(x,y,z)=\frac{1}{2}\int_0^{2\pi}\frac{\rho^2(\beta)}{(\rho(\beta)-s)^2}\int_{-\infty}^{\infty}R(p,\zeta,\beta)f\left(\frac{\rho(\beta)t}{\rho(\beta)-s}-p\right)\frac{\rho(\beta)}{\sqrt{\rho^2(\beta)+p^2+\zeta^2}}dpd\beta, \tag{2}$$

where

$$\zeta=\frac{\rho(\beta)(z-h(\beta))}{\rho(\beta)-s}.$$

It becomes clear that the essential step in Feldkamp-type cone-beam reconstruction is to modify cone-beam projection data so as to achieve exact transaxial reconstruction for any δ and *c* functions,. Correction is done by multiplying cone-beam data with the cosine of the X-ray tilting angle. Consequently, Feldkamp-type reconstruction can be decomposed into two steps: cone-beam to fan-beam data conversion and fan-beam reconstruction.

With arguments similar to those described above, cone-beam reocnstruction can also be achieved via correcting cone-beam data to fan-beam data in an inclined plane under the condition that a projected scanning locus stays outside a projected specimen support, the projection direction being defined by the normal of the tilited plane. It was previously established that the longitudinal integral of an reconstructed image volume by Feldkamp-type algorithms is exact (Feldkamp et al. 1984, Wang et al. 1992). We similarly proved that Feldkamp-type reconstruction with respect to a tilted longitudinal axis produces the exact 2-D parallel projection along the tilted longitudinal axis. This finding updates our earlier result that 2-D parallel-beam projections can be aproximately computed from cone-beam data (Lin et al. 1993). In practice, full 3-D information is often not useful, several stereo projection image pairs may be sufficient in some applications. Hence, exact 2-D parallel-beam projection pairs are the most desirable.

Our derivative-free noncircular fan-beam formula utilizes full-scan data, which consist of two complete projection data sets. Actually, fan-beam reconstruction can also be performed with either half-scan or double full-scan projection data. Accordingly, half-scan and double-helix-scan cone-beam algorithms can be formulated. The above discussion with one scanning turn can be extended to half- and double-helix-scan cases, respectively. With the same progection data correction, exact transaxial reconstruction can be achieved for $\delta(\vec{x}-\vec{x}_0)$ and $c(\vec{x})$ with half-or double-helix-scan data. In the half-scan case, the angular range involved in a transaxial slice reconstruction is substantially reduced. As a result, half-scan cone-beam reconstruction (Wang et al. 1994) may improve longitu dinaltemporal resolution. In the double-helix-scan case, a transaxial slice is reconstructed with consine-corrected and linearly combined projection data from twins of scanning turns. Double-helix-scan cone-beam reconstruction is exact for a specimen with linear longitudinal variation.

REFERENCES

Axelsson C, PE Danielsson. 1994. Three-dimensional reconstruction from cone-beam data in O ($N^3 \log^N$) time. Phys. Med. Biol. **39:** 477-491.

Cheng PC, TH Lin, G Wang, DM Shinozaki, HG Kim, SP Newberry. 1992. Review on the development of cone-beam X-ray microtomography. Proc. of the 13th International Congress on X-ray Optics and Microanalysis: 559-556, Manchester, UK.

Danielsson PE. 1992. From cone-beam projections to 3D Radon data in O ($N^3 \log^N$) time. Proc. IIEEE Medical Imaging Conference: 1135-1137.

Edholm PR, GT Herman. 1987. Linograms in image reconstruction from projections, IEEE Trans. Medical Imaging **6:** 301-307.

Feldkamp LA, LC Davis, JW Kress. 1984. Practical cone-beam algorithm. J. Opt. Soc. Am. **A1:** 612-619.

Grangeat P. 1991. Mathematical framework of cone-beam 3D reocnstruction via the first derivative of the Radon transform. Mathematical Methods in Tomography (Lecture Notes in Mathematics) ed. Herman GT, Louis AK and Natterer F.

Gullberg GT, GL Zeng, FL Datz, PE Christian, CH Tung, HT Morgan. 1992. Review of convergent beam tomography in single photon emission computed tomography. Phys. Med. Biool. **37:** 507-534.

Lin TH, G Wang, PC Cheng . 1993. Reconstruction of orthogonal views from cone-beam projections. Journal of Scanning Microscopy **15** (Suppl III): 61-63.

Smith BD. 1985. Image reconstruction from cone-beam projections: necessary and sufficient conditions and reconstruction methods. IEEE Trans. Medical Imaging **4:** 14-28.

Smith BD. 1990. Cone-beam tomography: recent advances and a tutorial review. Opt. Eng. **29:** 524-534.

Tuy HK. 1983. An inversion formula for cone-beam reconstruction. SIAM J. Appl. Math. **43:** 546-552.

Wang G, TH Lin, PC Cheng, DM Shinozaki, HG Kim. 1991. Scanning cone-beam reconstruction algorithms for X-ray microtomography. Proc. of SPIE **1556:** 99-113.

Wang G, TH Lin , PC Cheng, DM Shinozaki. 1992. Point spread function of the general cone-beam X-ray reconstruction formula. Journal of Scanning Microsocpy **14:** 187-193.

Wang G, TH Lin, PC Cheng. 1993a. A derivative-free non-circular fan-beam reconstruction formula. IEEE Trans. Image Processing **2:** 543-547.

Wang G, TH Lin, PC Cheng, DM Shinozaki. 1993b. A general cone-beam reconstruction algorithm. IEEE Trans. Medical Imaging **12:** 486-496.

Wang G, Y Liu, TH Lin, PC Cheng, DM Shinozaki. 1994. Half-scan cone-beam X-ray microtomography formula. Journal of Scanning Microscopy **26:** 216-220.

Wang G, PC Cheng. 1995. Rationale of Feldkamp-type cone-beam CT, Proc. of the 4th International Conference for Young Computer Scientists (in press).