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Technical Report

Maximum Volume Coverage in Spiral Computed Tomography Scanning

Ge Wang, PhD, and Michael W. Vannier, MD

Rationale and Objectives. Given a slice thickness and an image noise level, volume coverage in spiral computed tomography (CT) scanning was maximized with respect to detector collimation and table increment.

Methods. The maximization was analytically performed on the basis of the following relationships: (1) With the half-scan interpolation method, the slice sensitivity profile (SSP) in spiral CT scanning is the convolution of a rectangular detector response function (the width is equal to the detector collimation) and a triangular table motion function (the base is equal to the table increment); (2) the SSP variance is equal to the sum of squared detector collimation divided by 12 and squared table increment divided by 24; (3) the image noise variance is inversely proportional to tube current and detector collimation; and (4) the maximum continuous scanning time is inversely proportional to tube current.

Results. Given a limited tube heat capability, a slice thickness, and an image noise level, we proved that the maximum volume coverage is reached with a pitch of $\sqrt{2}$.

Conclusion. With a heat-limited tube and for a specified image quality in terms of slice thickness and image noise, a pitch of 1.4 should be used for maximum volume coverage in spiral CT scanning.

Key Words. Computed tomography; spiral-helical techniques; imaging protocols; image quality; volume coverage.

Spiral-helical X-ray computed tomography (CT) scanning is an important recent advance for volumetric imaging [1-6]. The original motivation for developing spiral CT scanning was to reduce motion artifacts. It has been established recently that for a given X-ray dose and with overlapping reconstruction, spiral CT scanning allows substantially better longitudinal bandwidth than conventional incremental CT scanning [7-9].

In spiral CT scanning, planar projection data sets are interpolated from spirally collected raw projection data. Among the various interpolation techniques, linear interpolation is usually preferred because of its efficiency and performance. Typical linear interpolation techniques include full scan with interpolation (FI), half scan with interpolation (HI), and half scan with extrapolation (HE) [6]. In the FI method, a set of planar projection data in a 360° angular range is obtained via linearly interpolating neighboring raw projection data at the same orientation, hence the raw data span a 720° angular range. The HI method uses redundancy of raw data, interpolates neighboring raw data at opposite directions, and involves only an angular range of 360° plus two fan angles. The HE method eliminates the condition required by the HI method that the opposite projection rays must be from different sides of a reconstruction plane. In the HE method, if the opposite rays

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originate from the same side of the plane, extrapolation is performed to estimate the corresponding projection value; otherwise, interpolation is done as in the HI method. Using the notations chosen by Kalender and Polacin [10] and Polacin et al. [11], the FI and HI methods are equivalent to the 360° linear interpolation and 180° linear interpolation methods, respectively. The HI and HE methods are widely used in practice.

Currently, spiral CT scanning has gained general acceptance as the standard medical CT mode of imaging. However, X-ray tube and dose considerations limit spiral CT volume coverage during a single scan [12–15] given slice thickness and image noise. The limited volume coverage may be smaller than structures of interest, resulting in the need for multiple spiral scans. Discontinuous data acquisition is not as good as single-breathhold scanning in terms of temporal image resolution. In this article, for a specified image quality in terms of slice thickness and image noise, we describe the maximization of spiral CT volume coverage by selecting a detector collimation, a table increment, and a scanning pitch, which is defined as the ratio of the table increment to the detector collimation.

MATERIALS AND METHODS

Our study was based on the model characterized by the following relationships regarding the slice sensitivity profile (SSP), slice thickness, image noise, and continuous scanning time.

Slice Sensitivity Profile

With the HI method, the SSP in spiral CT scanning is a convolution of a rectangular detector response function (the width is equal to the detector collimation) and a triangular table motion function (the base is equal to the table increment per gantry rotation; Appendix A).

Slice Thickness

The SSP variance is equal to the sum of squared detector collimation divided by 12 and squared table increment divided by 24 (Appendix B).

Image Noise

Projection noise variance is inversely proportional to the number of detected photons. As a result, image noise variance is approximately inversely proportional to the tube current and the detector collimation (Appendix C).

Continuous Scanning Time

The maximum continuous scanning time is inversely proportional to the tube current (Appendix C).

A key fact for maximizing the spiral CT volume coverage given slice thickness and image noise is that the total volume coverage during a spiral scan is proportional to both the table increment (T) and the detector collimation (D). It is evident that the volume coverage is proportional to T ; in other words, the greater T is, the larger the volume that will be covered. On the other hand, if D is increased, the tube current must be proportionally decreased to maintain the same photon statistics in order to achieve the same image noise level. As a result, the maximum continuous scanning time can be proportionally increased before reaching the tube heating limit. Because the volume coverage is proportional to the total scanning time, the volume coverage is proportional to D .

We prove in Appendix C that the maximum continuous scanning range is proportional to the SSP variance and image noise variance and that it is a convex function of the pitch. In Appendix D, we prove that the pitch function is maximized at a pitch of $\sqrt{2}$.

In other words, we link the maximum scanning coverage to three important parameters: the SSP variance (closely related to the slice thickness), the image noise variance, and the pitch (in the form of a pitch function). Specifically, when the scanning range is doubled, image quality must be correspondingly degraded in terms of either a doubled SSP variance or a doubled image noise variance. If both SSP variance and image noise variance are specified, the maximum volume coverage is reached with a 1.4 pitch. In Figure 1, the pitch function is plotted at typical pitch values. Note that the volume coverage is stable around the optimal pitch of $\sqrt{2}$.

RESULTS

Artifacts and Spatial Variation of SSP and Noise

Artifacts caused by high-pitch scanning and spatial variation of the SSP, and the image noise caused by spiral interpolation and reconstruction algorithms, were not considered in our model. These aspects are well understood. Stair-step artifacts in spiral CT images are associated with inclined surfaces in reformatted images, and they are more remarkable with greater pitches [16]. To generate spiral CT images with few stair-step artifacts, the table increment should not be much greater



FIGURE 1. Pitch function at typical pitch values. For a given slice thickness and an image noise level, the maximum volume coverage is reached at the pitch of 1.4.

than detector collimation, and both should be less than longitudinal dimensions of features on inclined surfaces. Strictly speaking, the SSP in spiral CT scanning is spatially variant [17]. In an extreme case, the change in the slice thickness is no more than 10%, and the skewness of the SSP is within 0.15 [17]. Spiral CT image noise is spatially invariant with the HI method in a parallel-beam projection model [18], as noted in Appendix C. The image noise values so predicted [18] are consistent with previously reported experimental and numeric results at the gantry center [6, 7, 19]. However, under a fan-beam projection model, there is a 40% image noise variation over a typical field of view [20].

In the first-order approximation, we omitted these complicated effects for the purpose of our study. Actually, it was these omissions and other approximations, including the omission of the noise correlation, that allowed us to obtain closed-form solutions, revealing fundamental interplays of key imaging and image-quality parameters.

Noise Correlation

Optimization is defined as the choice of parameters that maximize the performance. In CT scanning, this may be interpreted as the highest detectability of a target. It is known that the noise correlation in reconstructed spiral CT images changes with pitch. Regarding the detection of a target in differently correlated noisy environments, our consideration is as follows.

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From the viewpoint of pattern recognition, target detection can be interpreted as classification of an image into a target and a background. An approximate statistical model consists of two Gaussian probability distributions of the same variance that correspond to the target and the background, respectively. The M-distance is typically used to describe the separability between the classes. The M-distance is directly proportional to the difference between the means and inversely proportional to the variance, without any involvement of the noise correlation. The Bayesian classifier is the optimal for classification in this context. It has been proved that the Bayesian classification error can be expressed as a monotonic decreasing function of the M-distance, approaching zero in the limiting case. Various types of noise correlation manifest as different texture features but do not help improve classification in either two or three dimensions because these features are superimposed over the whole field of view. In visual image interpretation, the correlation length of noise is typically smaller than the dimensions of the structures of interest. Hence, noise variance is more important to detectability than is noise correlation. The noise correlation was only of secondary importance for target detection and was therefore omitted from our model.

HE and FI Methods

Although the formulation was done with the HI method, the conclusions also are valid for the HE method because the HI and the HE methods are highly similar in terms of the SSP at the isocenter and the image noise characteristics.

For better contrast resolution, the FI method is sometimes used in spiral CT scanning. It is known that with the FI method, the SSP is the convolution of the same rectangular detector response function and a triangular table motion function with its base being twice the table increment. Similarly, it can be proved that given slice thickness and image noise, the maximum volume coverage in this case can be reached with a pitch equal to $\frac{\sqrt{2}}{2}$.

In addition to HI, HE, and FI, there are other interpolation methods for spiral CT scanning [6]. In addition, the relationship between the maximum continuous scanning time and the tube current may be more complicated than what we modeled. However, using the method we describe here, we can solve the maximum volume coverage problem case by case.

Protocol Refinement

To apply our findings in practice, one may begin with a given spiral CT scanning protocol in which the maximum continuous scanning time is already in use. In such a protocol, the slice thickness can be computed using our slice thickness formula, and the image noise can be measured. To maintain the same slice thickness, the slice thickness formula becomes the constraint. Under this constraint, the pitch can be adjusted to $\sqrt{2}$. After the adjustment, to maintain the same image noise level, the tube current is set according to its inverse proportional relationship to the detector collimation. By doing so, one ends up with a longer scanning range and the same image quality in terms of slice thickness and image noise.

Note that what we have optimized are mathematically well-defined quantities. Their relevance to diagnostic performance needs further evaluation. For example, what the best indicator is for the slice thickness is still unanswered.

CONCLUSIONS

On the basis of our simplified spiral CT model, which consists of the SSP, the slice thickness, the image noise, and the maximum continuous scanning time formulas, we have proved that in spiral CT scanning with the HI method, the maximum volume coverage is reached at a pitch of $\sqrt{2}$ given a slice thickness and an image noise level.

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APPENDIX A

Slice Sensitivity Profile Formula

A longitudinal detector response function can be modeled as

$$f(z) = \frac{1}{D} \text{rect}\left(\frac{z}{D}\right), \quad (1)$$

where z is the longitudinal coordinate, D is the longitudinal dimension of the detector collimation, and $\text{rect}(\cdot)$ is the rectangular function

$$\text{rect}(z) = \begin{cases} 1, & z \in \left(-\frac{1}{2}, \frac{1}{2}\right) \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Because of the continuous table motion and resulting interpolation process, the slice sensitivity profile (SSP) in spiral computed tomography (CT) scanning is degraded compared with $f(z)$. If the half scan with interpolation method is used and the reconstruction interval is sufficiently small [7-9], the SSP in spiral CT scanning can be expressed as

$$p(z) = f(z) * g(z) = \int_{-\infty}^{\infty} f(t)g(z-t)dt, \quad (3)$$

where $g(z)$ is the table motion function defined as

$$g(z) = \begin{cases} \frac{2}{T} + \frac{4z}{T^2}, & z \in \left(-\frac{T}{2}, 0\right) \\ \frac{2}{T} - \frac{4z}{T^2}, & z \in \left(0, \frac{T}{2}\right) \\ 0, & \text{otherwise,} \end{cases} \quad (4)$$

where T denotes the table increment [7].

APPENDIX B

Slice Thickness Formula

There are various ways to quantify the narrowness of the slice sensitivity profile (SSP). In addition to the full width at half-maximum intensity (FWHM) and the full width at tenth maximum intensity (FWTM), which have been discussed frequently in the literature, we think that the standard deviation of the SSP also is a good alternative because the standard deviation is the standard statistic used to quantify the dispersion of a distribution. We used the standard deviation for a relative assessment of the spiral computed tomography (CT) SSP transverse variation [17]. The standard deviation is strongly correlated with both the FWHM and the FWTM. In the case of a Gaussian SSP $\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$, the full width at p -percentage-maximum intensity (FWpM) equals $2\sqrt{2}\sigma\sqrt{|\log p|}$. In other words, the standard deviation of the SSP is proportional to the FWpM. In this sense, the standard deviation and the FWpM are equivalent for quantifying the SSP narrowness and the slice thickness.

On the basis of the spiral CT SSP expressed in Appendix A, we have the following slice thickness formula:

$$\begin{aligned} \sigma^2 &= \int_{-\infty}^{\infty} z^2 p(z) dz \\ &= \int_{-\infty}^{\infty} f(t) \left[\int_{-\infty}^{\infty} z^2 g(z-t) dz \right] dt \\ &= \int_{-\infty}^{\infty} f(t) \left(\frac{T^2}{24} + t^2 \right) dt \\ &= \frac{D^2}{12} + \frac{T^2}{24}. \end{aligned} \quad (5)$$

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APPENDIX C

Volume Coverage Formula

To derive a volume coverage formula, in addition to the slice thickness formula derived in Appendix B, two more formulas are needed for the image noise and the maximum continuous scanning time, respectively.

Assuming that parallel-beam projection data are corrupted by an additive white noise, spiral computed tomography (CT) image noise variance with typical linear interpolation methods was obtained analytically [18]. One finds that the spiral CT image noise variance after the longitudinal integral is independent of the transaxial position and proportional to raw projection noise variance [18]. Because the projection noise variance is inversely proportional to the number of detected photons [21], the image noise variance, η^2 , is inversely proportional to the tube current, I , and the detector collimation, D :

$$\eta^2 = \frac{c_\eta}{ID}, \quad (6)$$

where c_η is a constant depending on the detector efficiency, the reconstruction filter, and so on.

Approximately, the maximum continuous scanning time, t , is inversely proportional to the tube current, I :

$$t = \frac{c_t}{I}, \quad (7)$$

where c_t is a constant, quantifying the tube heating limit.

We can now maximize the continuous scanning range, L , given the slice sensitivity profile (SSP) variance, σ^2 , and the image noise variance, η^2 .

Clearly,

$$L = Tt. \quad (8)$$

Given an image noise variance, η^2 , and using equations 6 and 7, we have

$$\begin{aligned} L &= T \frac{c_t}{I} \\ &= T \frac{c_t}{\frac{c_\eta}{\eta^2 D}} \\ &= \frac{c_t}{c_\eta} \eta^2 D T \\ &= c \eta^2 D T, \end{aligned} \quad (9)$$

where $c = \frac{c_l}{c_n}$.

Given an SSP variance, σ^2 , and using equation 5, we have

$$L = c\eta^2 D \sqrt{24\left(\sigma^2 - \frac{D^2}{12}\right)}, \quad (10)$$

where $D \in (0, 12\sigma^2)$.

For a given SSP variance, σ^2 , the pitch, p , can be expressed as a function of D :

$$\begin{aligned} p^2 &= \frac{T^2}{D^2} \\ &= \frac{24}{D^2} \left(\sigma^2 - \frac{D^2}{12} \right) \end{aligned} \quad (11)$$

Solving this equation for D^2 , we have

$$D^2 = \frac{24\sigma^2}{p^2 + 2}. \quad (12)$$

Substituting equation 12 into equation 10, we obtain

$$L = 24c\sigma^2\eta^2 g(p), \quad (13)$$

where $g(p) = \frac{1}{\sqrt{p^2+2}} \left(1 - \frac{2}{p^2+2} \right)$ is a convex function of the pitch, $p \in (0, \infty)$.

APPENDIX D

Optimal Pitch

To maximize the volume coverage in spiral computed tomography scanning subject to given σ^2 and η^2 is to maximize $g(p)$ with respect to p .

Let

$$\frac{dg^2(p)}{dp} = 0. \quad (14)$$

We then get

$$-\frac{2p}{(p^2+2)^2} \left(1 - \frac{2}{p^2+2} \right) + \frac{1}{(p^2+2)(p^2+2)^2} = 0. \quad (15)$$

Therefore, $p = \sqrt{2}$.

Announcements

The University of Michigan Medical School, Department of Radiology, is sponsoring the **10th Annual Symposium on Breast Disease: Diagnostic Imaging and Current Management** which will be held July 14-16, 1996, at the Grand Hotel on Mackinac Island, MI. Fifteen hours of Category 1 credit of the Physician's Recognition Award of the American Medical Association to be announced. Application for credit also has been submitted to the American Osteopathic Association. Other credits by specialty may apply. The course director is Dorit D. Adler, MD.

For more information, contact the Registrar, Towsley Center for Continuing Medical Education, Department of Postgraduate Medicine and Health Care Professions, University of Michigan Medical School, P.O. Box 1157, Ann Arbor, MI 48106-1157; (313) 763-1400, fax (313) 936-1641.

Radiology International is sponsoring the 18th International Symposium, **Radiology in Bamberg**, September 28, 1996, through October 6, 1996, in Bamberg, Germany. Thirty-five hours of Category I continuing medical education credits will be awarded, including 5 hours of MR imaging of bone and joints, CT/MR imaging of the head and neck, helical CT, and color flow Doppler. The program directors are Beatty Crawford, MD, and Betsy Birchenough. Scientific registration is \$700.

For more information, contact Betsy Birchenough, Director, P.O. Box 95, 340 Cleverdale Rd., Cleverdale, NY 12820; (518) 656-3104, fax (518) 656-9815.