Optimal pitch in spiral computed tomography

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The focus of this paper is to analytically optimize spiral/helical computed tomography (CT) protocols based on a simplified imaging model. Spiral CT was approximately modeled as follows: Using the half-scan raw data interpolation method, the variance of the spiral CT slice sensitivity profile is equal to the sum of squared detector collimation divided by 12 and squared table increment divided by 24. Image noise variance is inversely proportional to tube current and detector collimation. The maximum continuous scanning time is inversely proportional to tube current. Slice thickness, image noise, and signal-to-noise ratio were, respectively, optimized for a given scanning coverage, consistently resulting in pitch of square root of 2. To avoid longitudinal aliasing, at least 2–3 transverse slices should be reconstructed per collimation. When the simplified spiral CT model is valid and a scanning range specified, 1.4 pitch is required for optimal image quality. The method can be applied to more accurate spiral CT models. [S0094-2405(97)00610-X]

Key words: computed tomography (CT), spiral/helical technique, imaging protocols, image quality, optimization

I. INTRODUCTION

Spiral/helical computer tomography (CT) is an important recent advance for volumetric imaging.[1–5] In spiral CT, specification of multiple acquisition and reconstruction parameters is required. The unit scanning pitch is popular in practice. Although use of pitches greater than 1 were reported,[6–10] the theoretical foundation has never been addressed.

In this paper, we approximately model the interaction between imaging parameters (detector collimation, table increment, tube current and heat capability, reconstructed slice number per collimation) and image characteristics (slice thickness, image noise, and signal-to-noise ratio), and optimize imaging protocols for a given scanning coverage with respect to slice thickness, image noise, and signal-to-noise ratio, respectively.

II. MATERIALS AND METHODS

Spiral CT is approximately modeled by the formulas in this section. The notations and definitions are given below, where the unit of D, T, and L can be either cm or mm.

D: *Longitudinal dimension of the detector collimation,*

$\mathrm{rect}(z)$: Rectangular function:

$$\mathrm{rect}(z)=\begin{cases}1, & z\in[-\frac{1}{2},\frac{1}{2}),\\ 0, & \text{otherwise,}\end{cases}$$

$f(z)$: Longitudinal detector response function;

$$f(z)=\frac{1}{D}\,\mathrm{rect}\left(\frac{z}{D}\right),$$

T: Table increment per gantry rotation;

$g(z)$: Weighting function due to table motion and half-scan interpolation (180° LI) (Ref. 11):

$$g(z)=\begin{cases}\frac{2}{T}+\frac{4z}{T^2}, & z\in\left[-\frac{T}{2},0\right),\\ \frac{2}{T}-\frac{4z}{T^2}, & z\in\left[0,\frac{T}{2}\right),\\ 0, & \text{otherwise,}\end{cases}$$

p: Pitch:

$$p=\frac{T}{D},$$

σ: Slice thickness, defined as the standard deviation of the slice sensitivity profile (SSP),

η: Standard deviation of image noise,

I: Tube current,

t: Maximum continuous scanning time,

L: Maximum continuous scanning coverage,

Δz: Transaxial reconstruction interval,

n: Reconstructed slice number per collimation.

A. SSP

We assume that the most popular half-scan raw data interpolation method[11] is employed, and transaxial reconstruction interval Δz sufficiently small, the SSP in spiral CT is[4]

$$p(z)=f(z)*g(z)=\int_{-\infty}^{\infty}f(t)g(z-t)dt. \quad (1)$$

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TABLE I. Recommended number of reconstructed slices per collimation.

p	0.1	0.3	0.5	0.7	0.9	1.1	1.3	1.5	1.7	1.9
n	3.30	3.12	3.12	2.97	2.79	2.61	2.44	2.27	2.12	1.98

Geometrically, the SSP is a convolution of a rectangular detector response function and a triangular table motion function. The bases of the rectangular and triangular functions are equal to detector collimation D and table increment T, respectively.

B. Slice thickness

Using Formula (1), the following spiral CT slice thickness formula can be obtained (Appendix A):

$$\sigma^2=\frac{D^2}{12}+\frac{T^2}{24}. \tag{2}$$

C. Image noise

Assume that parallel-beam projection data are corrupted by an additive white noise, spiral CT image noise variance with typical linear interpolation methods was analytically obtained.[12] It was found that the spiral CT image noise variance after longitudinal integral is independent of the transaxial position and proportional to the raw projection noise variance.[12] The theoretical findings are consistent with previously reported experimental and numerical results at the gantry center.[3,11–13] Since projection noise variance is inversely proportional to the number of detected photons,[14] image noise variance is inversely proportional to tube current and detector collimation:

$$\eta^2=\frac{c_\eta}{ID}, \tag{3}$$

where c_η is a constant depending on detector efficiency, reconstruction filter, and other factors.

D. Maximum scanning time and range

Roughly, the maximum continuous scanning time is inversely proportional to the tube current:

$$t=\frac{c_t}{I}, \tag{4}$$

where c_t is a constant, quantifying the tube heat limit. Consequently, the maximum continuous scanning range is

$$L=c_L Tt, \tag{5}$$

where c_L is a constant.

III. RESULTS

A. Optimal pitch

Formulas (2), (3), (4), and (5) together describe interplays of imaging and image quality parameters. Under these constraints, optimization can be analytically performed with respect to slice thickness, image noise, and signal-to-noise ratio, given a prespecified scanning coverage.

Mathematically, we have the following three optimization problems, which are clinically significant:

(1) Given a scanning coverage L and an image noise level η, minimize the slice thickness σ.
(2) Given a scanning coverage L and a slice thickness σ, minimize the image noise level η.
(3) Given a scanning coverage L, minimize the product of slice thickness and image noise, which is equivalent to minimize $\sigma^2\eta^2$, so that the conflicting requirements for slice thickness and image noise can be optimally balanced.

Actually, the third optimization problem is also to maximize the signal-to-noise ratio in the sense that the signal strength is measured as being proportional to $\frac{1}{\sigma}$ and the noise as proportional to η. The rationale for quantifying the signal strength in this manner can be explained as follows. Since our SSP profile contains a unit area, the smaller σ is, the narrower the SSP, and the greater the peak of the SSP. Clearly, the peak of the SSP can be used to represent the signal strength.

Interestingly, in all the three cases the optimal pitch is $\sqrt{2}$ (Appendix B). With the pitch being set to $\sqrt{2}$, other imaging parameters, such as detector collimation and table increment, can be determined in specific applications.

B. Overlapping reconstruction

The standard deviation of the SSP can be computed according to Formula (2). If the SSP is approximated as a Gaussian distribution, then the standard deviation of the modulation transfer function (MTF, that is, the Fourier transform of the SSP) can be derived. Further assuming that the cutoff frequency of the MTF is given by tripling the Gaussian MTF standard deviation, we can then find the minimum longitudinal sampling step Δz by meeting the Nyquist sampling requirement. As a result, we have the following formula (Appendix C) for the minimum reconstructed slice number n per collimation D:

$$n=\frac{D}{\Delta z}=\frac{3}{\pi\sqrt{1/12+p^2/24}}. \tag{6}$$

Typical p and corresponding n values are given in Table I.

IV. DISCUSSION AND CONCLUSION

The findings reported here are obtained based on the idealized mathematical model specified by Formulas (2), (3), (4), and (5). They should be used with caution. There are a number of approximations and limitations in our model. If any of these is invalid in intended applications, our conclusions may not hold.

First, besides the half-scan interpolation method,[11] other spiral CT raw data interpolation methods are also used in practice.[3] Generally, the specific form of the weighting function $g(z)$ is determined by the detail of a linear spiral CT raw data interpolation method, so is the optimal pitch. For better low-contrast resolution, full-scan interpolation ($360°$ LI)[11] is sometimes used in spiral CT. With the full-scan interpolation method, the SSP is the convolution of the detector response function and a triangular weighting function whose base equals doubled table increment. It can be similarly proved that in this case the optimal pitch is $\sqrt{2}/2$.

Second, the detector response may not be a rectangular function, if detector collimation is small. In this case, the detector response function can be experimentally measured, and converted to the equivalent rectangular function that has the same variance as the experimentally measured detector response function. Using the width of the equivalent rectangular detector response as detector collimation, the above derivations and corresponding conclusions are still valid.

Third, the tube heat capacity depends on tube voltage, and is a nonlinear function of tube current. Equation (4) as a linear approximation is quite rough. The exact function of the tube heat capacity may be obtained by fitting data from manufacturers, and used to refine the optimization.

Fourth, from a practical application standpoint, one critical criterion for selection of pitch is the presence and severity of image artifacts, such as stair-step artifacts.[15] Because these artifacts have not been analytically modeled yet, they were neglected in our current work. Furthermore, it is generally believed that for a pitch less than 2, image artifacts are not a serious problem. Strictly speaking, if artifacts are taken into account, the optimization results may differ.

Fifth, modeling the SSP via a Gaussian distribution may produce substantial errors. A more precise way to estimate the slice interval for overlapping reconstruction is to derive the MTF directly, and then apply the Nyquist sampling criterion. However, our simplified treatment is needed to clearly reveal the major part of the interaction.

Also, what we optimized are mathematically well-defined quantities. Their relevance to diagnostic performance needs being further evaluated in various applications. The well-established ROC approach can be used for statistical significance.

In conclusion, given a continuous scanning range, the optimal pitch is $\sqrt{2}$ for either minimizing slice thickness, minimizing image noise or maximizing signal-to-noise ratio, provided that our spiral CT model is satisfied. To avoid aliasing the MTF, about 2–3 transverse slices should be reconstructed per collimation. Although our work has several above mentioned weaknesses, the results seem consistent with the empirical choice of pitch, and the method can be used in further studies.

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APPENDIX A: SLICE THICKNESS FORMULA

The way to quantify the slice thickness from the SSP is not unique. In addition to the full-width-at-half-maximum (FWHM) and the full-width-at-the-tenth-maximum (FWTM), the standard deviation of the SSP is also a good alternative, because the use of the standard deviation is the standard statistical method to quantify the dispersion of a distribution. We used the standard deviation for a relative assessment of spiral CT SSP transverse variation.[16] The standard deviation is strongly correlated with either the FWHM or the FWTM. In the case of an Gaussian SSP $(1/\sqrt{2\pi\sigma})e^{-z^2/2\sigma^2}$, the full-width-at-$p$-percentage-maximum equals $2\sqrt{2}\sigma\sqrt{|\log p|}$. Therefore, a given longitudinal CT image resolution can be translated into a specific SSP standard deviation σ.

Using Formula (1),[17]

$$\begin{aligned}\sigma^2 &= \int_{-\infty}^{\infty} z^2 p(z)dz \\ &= \int_{-\infty}^{\infty} f(t)\left[\int_{-\infty}^{\infty} z^2 g(z-t)dz\right]dt \\ &= \int_{-\infty}^{\infty} f(t)\left[\frac{T^2}{24}+t^2\right]dt \\ &= \frac{D^2}{12}+\frac{T^2}{24}. \end{aligned} \quad \text{(A1)}$$

APPENDIX B: OPTIMIZATION

In what follows L is always assumed to be a prefixed constant.

A. Minimize σ given η

Using Formulas (3), (4), and (5), we have

$$\begin{aligned} T &= \frac{L}{t} = \frac{L}{c_t/I} \\ &= \frac{L}{(c_t/c_\eta)/\eta^2 D} = \frac{c_\eta L}{c_t \eta^2 D} = \frac{c}{D}, \end{aligned} \quad \text{(B1)}$$

where $c = c_\eta L/c_t\eta^2$ is a constant. Substituting the above equation into Formula (2),

$$\sigma^2(D^2) = \frac{D^2}{12} + \frac{c^2}{24D^2}. \quad \text{(B2)}$$

Consequently,

$$\frac{d\sigma^2(D^2)}{dD^2}=\frac{1}{12}-\frac{c^2}{24D^4}. \tag{B3}$$

Let

$$\frac{d\sigma^2(D^2)}{dD^2}=0; \tag{B4}$$

we have

$$D^2=\frac{c}{\sqrt{2}}. \tag{B5}$$

Therefore,

$$p=\frac{T}{D}=\sqrt{2}. \tag{B6}$$

B. Minimize η given σ

Using Formulas (3), (4), and (5), we have

$$\eta^2=\frac{c_\eta}{ID}=\frac{c_\eta}{(c_t/t)D}=\frac{c_\eta}{(c_t/L)DT}=\frac{c}{DT}, \tag{B7}$$

where $c=c_\eta L/c_t$. Hence, under the constraint of Formula (2), the minimization problem is equivalent to maximize

$$f(D^2)=D^2T^2=24D^2\left(\sigma^2-\frac{D^2}{12}\right)=24\left(\sigma^2D^2-\frac{D^4}{12}\right). \tag{B8}$$

Clearly,

$$\frac{df(D^2)}{dD^2}=24\left(\sigma^2-\frac{D^2}{6}\right). \tag{B9}$$

Let

$$\frac{df(D^2)}{dD^2}=0; \tag{B10}$$

we have

$$D^2=6\sigma^2. \tag{B11}$$

Also,

$$T^2=24\left(\sigma^2-\frac{D^2}{12}\right)=12\sigma^2. \tag{B12}$$

That is,

$$p=\frac{T}{D}=\sqrt{2}. \tag{B13}$$

C. Maximize signal-to-noise ratio

We now minimize the product $\sigma^2\eta^2$. Using Formulas (3), (4), and (5) again,

$$\eta^2=\frac{c_\eta}{ID}=\frac{c_\eta}{(c_t/t)D}=\frac{c_\eta}{(c_t/L)DT}=\frac{c}{DT}, \tag{B14}$$

where $c=c_\eta L/c_t$. Using Formula (2),

$$f(p)=\sigma^2\eta^2=c\,\frac{D^2/12+T^2/24}{DT}=c\left(\frac{1}{12p}+\frac{p}{24}\right); \tag{B15}$$

$$\frac{df(p)}{dp}=c\left(-\frac{1}{12p^2}+\frac{1}{24}\right). \tag{B16}$$

Let

$$\frac{df(p)}{dp}=0; \tag{B17}$$

we have again

$$p=\sqrt{2}. \tag{B18}$$

APPENDIX C: OVERLAPPING RECONSTRUCTION

The standard deviation of the SSP can be computed according to Formula (2). Let us approximate the SSP as a Gaussian distribution. The standard deviation Σ of the Gaussian MTF (the Fourier transform of the Gaussian SSP) can be directly derived:

$$\Sigma=\frac{1}{2\pi\sigma}=\frac{1}{2\pi\sqrt{D^2/12+T^2/24}}. \tag{C1}$$

Further assuming that the MTF cutoff frequency $f_{\max}$ is given by tripling Σ, we have

$$f_{\max}=3\Sigma=\frac{3}{2\pi\sqrt{D^2/12+T^2/24}}. \tag{C2}$$

Hence, the longitudinal sampling step Δz, that is, the reconstruction interval between adjacent transverse slices, can be estimated by meeting the Nyquist sampling requirement:

$$\Delta z=\frac{1}{2f_{\max}}=\frac{\pi}{3}\sqrt{\frac{D^2}{12}+\frac{T^2}{24}}. \tag{C3}$$

In other words, the reconstructed slice number n per collimation D is:

$$n=\frac{D}{\Delta z}=\frac{3}{\pi\sqrt{1/12+p^2/24}}. \tag{C4}$$

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