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Differential phase-contrast interior tomography

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Abstract

Differential phase-contrast interior tomography allows reconstruction of a refractive index distribution over a region of interest (ROI) for visualization and analysis of structures inside a large biological specimen. In the imaging mode, x-ray scanning only targets an ROI in an object and a narrow beam passes through the object, allowing a significant reduction of both radiation dose and system cost. Inspired by recently developed compressive sensing theory, in a numerical analysis framework we show that accurate interior reconstruction can be achieved on an ROI from truncated differential projection data through the ROI via the total variation minimization, assuming a piecewise constant distribution of the refractive indices in the ROI. Then, we develop a practical iterative algorithm for such an interior reconstruction and perform numerical experiments to demonstrate the feasibility of the proposed approach.

(Some figures may appear in colour only in the online journal)

1. Introduction

X-ray computed tomography (CT) uses attenuation coefficients of an object to define interior structures. However, biological soft tissues in preclinical and clinical applications mainly consist of atoms of low atomic numbers, such as hydrogen, carbon, nitrogen and oxygen, and their elemental composition is nearly uniform. The attenuation differences among different light elements in biological tissues are quite small, and tissue characteristics are indiscernible in x-ray CT images. On the other hand, x-ray phase-contrast imaging utilizes refractive indices of an object to improve image quality significantly for a wide range of biological and medical studies (Momose \textit{et al} 1996, Momose 2003, Weitkamp \textit{et al} 2005).

The refractive index can be expressed in complex form: $n(\mathbf{r}) = 1 - \delta(\mathbf{r}) + i\beta(\mathbf{r})$, where $\delta(\mathbf{r})$ denotes the refractive index that is related to a phase shift, and $\beta(\mathbf{r})$ characterizes the attenuation properties of an object. The interaction cross section of the phase shift is about 1000 times larger than that of the linear attenuation for an x-ray energy range of 10–100 keV, which implies that phase-contrast imaging has a much higher sensitivity for light elements.
than conventional attenuation-contrast imaging. Therefore, x-ray phase-contrast imaging can be used to observe subtle structural variations of biological soft tissues (Momose et al 2006, Pfeiffer et al 2006).

The x-ray grating interferometer is a recently established phase-contrast imaging technique, which is used to extract the differential phase shift information at each projection view from moiré fringe patterns. In practice, x-ray gratings have a microscale period and high aspect ratio. It is rather difficult to make large size gratings, and acquired projection data may only cover a small portion of an object. Hence, an interior tomographic imaging mode would be valuable to reconstruct a refractive index distribution over a region of interest (ROI) for visualization and analysis of internal structures inside a large biological specimen, an animal or a patient.

In the interior imaging mode, x-ray scanning only targets an ROI in an object, and a narrow beam passes through the object, allowing a significant reduction of both radiation dose and system cost. The interior problem of attenuation-based CT has been actively studied over the past several years. The interior problem does not have a unique solution only from truncated projection data in an unconstrained setting Natterer 2001. Interestingly, recent results show that the interior problem is solvable if appropriate prior information is available. In particular, if the attenuation coefficient distribution on a small subregion in an ROI is known (Defrise et al 2006, Ye et al 2007, Coudurier et al 2008), or the attenuation coefficient distribution on an ROI has some characteristic, such as being piecewise constant or polynomial, the interior reconstruction has a unique solution (Yu and Wang 2009, Han et al 2009, Yang et al 2010).

Differential phase-contrast interior tomography, which we propose here, is to reconstruct a refractive index distribution over an ROI from truncated differential phase shift data in a theoretically exact fashion. The theory and techniques of attenuation-based interior tomography cannot be directly applied for differential phase-contrast interior tomography, because differential phase shift data measured by an x-ray grating interferometer are truncated differential projection data of refractive indices instead of projection data themselves. Anastasio and Pan (2007) presented a backprojection filtration algorithm for differential phase-contrast interior imaging. Pfeiffer et al (2008) reported a filtered backprojection (FBP) method for grating-based differential phase-contrast interior imaging. However, their results were not intended for theoretically exact reconstruction.

In this paper, we demonstrate that the differential phase-contrast interior problem has a unique solution if an underlying x-ray refractive index distribution is piecewise polynomial in an ROI. We also propose a practical iterative method to reconstruct a refractive index distribution over an ROI via the total variation (TV) minimization from truncated differential projection data through the ROI. Finally, we discuss relevant issues and conclude the paper.

2. Differential phase-contrast interior tomography theory

In grating interferometric imaging or diffraction enhanced imaging, an object is generally scanned in parallel-beam geometry. When an x-ray beam passes through an object, phase shift data can be expressed as a projection of the refractive index distribution \( \delta(\mathbf{r}) \) in the object (Paganin 2006):

\[
R[\delta(s, \theta)] = \frac{2\pi}{\lambda} \int_{R} \delta(s\theta + t\mathbf{\theta}_\perp) \, dt,
\]

where \( \lambda \) is the x-ray wavelength, and \( \theta \) a directional vector of the x-ray beam at a projection angle \( \alpha \), \( \theta = (\cos \alpha, \sin \alpha) \). In the context of x-ray grating interferometric imaging or diffraction enhanced imaging, one can only extract differential phase shift data
\(\partial R[\delta(s, \theta)]\) from measured moiré fringe patterns, which are differential projection data of the underlying refractive index distribution. Without loss of generality, we consider the 2D case of differential phase-contrast interior tomography. Suppose that an object is supported on a disk \(\Omega_1 = \{ r = (x, y) \in \mathbb{R}^2 : |r| \leq A \}\), and an interior ROI defined as \(\Omega_a = \{ r = (x, y) \in \mathbb{R}^2 : |r| < a \}\) for \(0 < a < A\).

Using a methodology similar to attenuation-based interior tomography (Yang et al 2010), we can obtain the following lemma 1.

**Lemma 1.** If \(h(r)\) and \(\delta(r)\) are piecewise smooth for \(r \in \mathbb{R}^2\), compactly supported on the disk \(\Omega_1\), and have the same differential projection data \(\partial R[h(s, \theta)] = \partial R[\delta(s, \theta)]\), \(-a < s \leq a, \theta \in S^1\), then the difference function \(u(r) = h(r) - \delta(r)\) is analytic in the disk \(\Omega_a\), and \(\partial R[u(s, \theta)] = 0, -a < s \leq a, \theta \in S^1\).

**Proof.** Let \(\Omega = (C \setminus \mathbb{R}) \cup (-a, a)\), \(g(x) = Hu(x)\) for \(x \in (-\infty, \infty)\), and define
\[
f(x) = -\frac{1}{\pi} \left( \int_{-\infty}^{-a} g(t) \, dt + \int_{a}^{\infty} \frac{g(t)}{z-t} \, dt \right) \quad \text{for} \quad z \in \Omega.
\]
The function \(f(z)\) is analytic in \(\Omega\). Let \(z = x + iy\) with \(y > 0\), and we have the following formulae from equation (2):
\[
\text{Re}[f(z)] = \frac{1}{\pi} \int_{R} \frac{-x-t}{(x-t)^2 + y^2} g(t) \, dt \quad \text{and} \quad \text{Im}[f(z)] = \frac{1}{\pi} \int_{R} \frac{-y}{(x-t)^2 + y^2} g(t) \, dt.
\]
From \(f(z) - f(\bar{z}) = 2\text{Im}[f(z)]\), we obtain (Titchmarsh 1948, Courdurier 2007)
\[
\lim_{y \to 0^+} \left[ f(z) - f(\bar{z}) \right] = \frac{1}{2i} g(x) \text{ is almost everywhere.}
\]
Since \(u(x)\) is polynomial and \(Hu(x) = 0\) for \(x \in (-a, a)\), function \(f(z)\) is polynomial on \((-a, a)\). Hence, we have that \(f(z)\) is polynomial in \(\Omega\). Hence, we have \(g(x) = 0\) almost everywhere from equation (4), which implies \(u(x) = 0\) almost everywhere.

**Lemma 2.** If a function \(u(x)\) is polynomial and \(Hu(x) = 0\) for \(x \in (-a, a)\), where \(Hu(x)\) is the Hilbert transform of \(u(x)\), then \(u(x) = 0\) almost everywhere.

**Proof.** Let \(\Omega = (C \setminus \mathbb{R}) \cup (-a, a)\), \(g(x) = Hu(x)\) for \(x \in (-\infty, \infty)\), and define
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f(x) = -\frac{1}{\pi} \left( \int_{-\infty}^{-a} g(t) \, dt + \int_{a}^{\infty} \frac{g(t)}{z-t} \, dt \right) \quad \text{for} \quad z \in \Omega.
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**Lemma 3.** If an image \(u(r)\) satisfies: (A) \(u(r) = p(r)\) for \(r \in \Omega_a\), where \(p(r)\) is polynomial, and (B) \(\partial R[u(s, \theta)] = 0, -a < s \leq a, \theta \in S^1\), then \(u(r) = 0\).

**Proof.** For an arbitrary \(\varphi_0 \in [0, \pi)\), let \(L_{00}\) be the line through the origin and tilted by \(\theta_0 = (\cos \varphi_0, \sin \varphi_0)\). When \(u(r)\) is restricted to the line \(L_{00}\), it can be expressed as
\[
v_{\varphi_0}(t) = u(t \cos \varphi_0, t \sin \varphi_0), t \in (-\infty, \infty).
\]
By the relationship between the backprojection of differentiated projection data and the Hilbert transform of an image (Yang et al 2010, Noo et al 2007), we have
\[
Hu_{\varphi_0}(t) = -\frac{1}{2\pi} \int_{\varphi_0}{}^{\varphi_0 + \frac{\pi}{2}} \partial R[u(s, \theta)]|_{s=r, \theta} \, d\varphi,
\]
where \(r = (t \cos \varphi_0, t \sin \varphi_0)\) and \(\theta = (\cos \varphi, \sin \varphi)\). By condition (A) and equation (5), we have \(v_{\varphi_0}(t) = p(t \cos \varphi_0, t \sin \varphi_0), t \in (-a, a)\), where \(p(t \cos \varphi_0, t \sin \varphi_0)\) is polynomial with respect to \(t\). By condition (B) and equation (6), we have
\[
Hu_{\varphi_0}(t) = 0 \text{ for } t \in (-a, a).
\]
By lemma 2, we have that \(v_{\varphi_0}(t) = 0\), which implies that \(u(r) = 0\) almost everywhere.

From lemma 3, we obtain the following theorem.
Theorem 1. Suppose that a refractive index image $\delta(r)$ of an object is piecewise polynomial in $\Omega_a$. If another refractive index image $h(r)$ is also piecewise polynomial in $\Omega_a$ and has the same differential projection data as $\delta(r)$, $\delta_s R [h(s, \theta)] = \delta_s R [\delta(s, \theta)]$ for $s \in (-a, a), \theta \in S^1$, then $h(r) = \delta(r)$.

As a special case, from lemma 3 we obtain the following corollary 1.

Corollary 1. If a refractive index image $\delta(r)$ is known on a small subregion $\Omega_0$ in an ROI, then the refractive index function can be uniquely determined from the truncated differential phase shift data through the ROI and the prior knowledge on $\Omega_0$.

Theorem 1 shows that the interior refractive index reconstruction from truncated differential projection data has a unique solution within the class of piecewise polynomial functions in $\Omega_a$. Next, let us present a practical method for interior refractive index reconstruction. In practical numerical computation, we commonly discretize $\Omega_a$ into a numerical mesh; for example, dividing a region $\Omega_a$ into rectangular elements: $G = \{E_k, k = 1, 2, \ldots, N\}$ such that $\Omega_a \subset \bigcup_{k=1}^{N} E_k$. Based on the numerical discretization of $\Omega_a$, the TV of a function $h(x, y)$ is defined by

$$TV(h) = \sum_{k=1}^{N} D_{E_k}(h),$$

where $D_{E_k}(h)$ represents an anisotropic norm

$$D_{E_k}(h) = |h(x_i, y_i) - h(x_i + d, y_j)| + |h(x_i, y_i) - h(x_i, y_i + d)|,$$

or an isotropic norm

$$D_{E_k}(h) = \sqrt{|h(x_i, y_i) - h(x_i + d, y_j)|^2 + |h(x_i, y_i) - h(x_i, y_i + d)|^2},$$

where $(x_i, y_i)$, $(x_i + d, y_j)$, and $(x_i, y_i + d)$ are nodal coordinates of $E_k$ with a mesh size of $d$. From the definition of TV, we have the following theorem.

Theorem 2. For a numerical mesh with a sufficiently small element size that discretizes $\Omega_a$, the TV of a piecewise constant refractive index function in $\Omega_a$ is smaller than that of any other functions if they give the same truncated differential projection data through $\Omega_a$.

Proof. Let $\delta(r)$ be a piecewise constant function in $\Omega_a$ and its truncated differential projection data are $\delta_s R [\delta(s, \theta)]$ for $s \in (-a, a), \theta \in S^1$. We assume that $\Omega_a$ contains a finite number of subsets, $\Omega_a = \bigcup_{m=1}^{M} \Omega_m$, with $\delta(r) = c_m$ for $r \in \Omega_m, m = 1, 2, \ldots, M$. We denote the boundary by subregions $\Omega_a$ and $\Omega_i$ as $\Gamma_i = \Omega_i \cap \Omega_i$. We separate the set $G$ into two parts, $G = B \cup C$, where $B$ only includes rectangular elements which interact with the boundary set $\{\Gamma_m\}$ in region $\Omega_a$, and $C$ covers rectangles which are inside subsets $\Omega_m, m = 1, 2, \ldots, M$. For another function $h(r)$, which has the same truncated differential projection data as $\delta(r)$, we have

$$\sum_{E_k \in G} D_{E_k}(h) = \sum_{E_k \in B} D_{E_k}(h) + \sum_{E_k \in C} D_{E_k}(h).$$

Let $u(r) = h(r) - \delta(r)$. Because $\delta(r)$ is piecewise constant in $\Omega_a$, we have

$$\sum_{E_k \in C} D_{E_k}(h) = \sum_{E_k \in C} D_{E_k}(u)$$

and

$$\sum_{E_k \in G} D_{E_k}(\delta) = \sum_{E_k \in B} D_{E_k}(\delta).$$
By the triangle inequality, we have
\[ \sum_{E_k \in B} D_{E_k}(\delta) = \sum_{E_k \in B} D_{E_k}(u) \leq \sum_{E_k \in B} D_{E_k}(h). \] (14)

From lemma 1, \( u(\mathbf{r}) \) is analytic in \( \Omega_a \). Clearly, the measure of \( B \) will decrease and the measure of \( C \) will increase when the elemental size of the numerical mesh is reduced. Hence, for a sufficiently small elemental size of the numerical mesh, the following inequality holds:
\[ \sum_{E_k \in C} D_{E_k}(u) - \sum_{E_k \in B} D_{E_k}(u) \geq 0. \] (15)

From equations (11), (12), (14), and (15), we obtain
\[ \sum_{E_k \in B} D_{E_k}(\delta) \leq \sum_{E_k \in G} D_{E_k}(h). \] (16)

From equations (13) and (16), we immediately obtain the theorem. \( \square \)

Furthermore, in the same spirit the above proofs, theorem 2 can be easily extended to following piecewise linear or polynomial refractive index distribution in an ROI.

**Corollary 2.** For a sufficiently small elemental size of a numerical mesh that discretizes \( \Omega_a \), the second-order TV of a piecewise linear refractive index function in \( \Omega_a \) is smaller than that of any other functions, if they have the same truncated differential projection data through the ROI, where the second-order TV of a function \( h(\mathbf{r}) \) is defined in anisotropic form by
\[
\text{TV}_2(h) = \sum_{i=1}^{M} \sum_{j=1}^{N} [(h(x_i - d, y_j) - 2h(x_i, y_j) + h(x_i + d, y_j))
+ (h(x_i, y_j - d) - 2h(x_i, y_j) + h(x_i, y_j + d))],
\]
or isotropic form by
\[
\text{TV}_2(h) = \sum_{i=1}^{M} \sum_{j=1}^{N} [(h(x_i - d, y_j) - 2h(x_i, y_j) + h(x_i + d, y_j)]^2
+ [h(x_i, y_j - d) - 2h(x_i, y_j) + h(x_i, y_j + d)]^2)^{1/2},
\]
where \((x_k - d, y_k), (x_k + d, y_k), (x_k, y_k - d), \) and \((x_k, y_k + d)\) are neighboring nodes of \((x_k, y_k)\) in the numerical mesh with an elemental size of \( d \).

Theorem 2 and corollary 2 give a practical numerical scheme for interior reconstruction, and show that the piecewise constant or piecewise polynomial function in an ROI is an optimal solution of the TV or high-order TV minimization subject to the constraints of truncated differential projection data through an ROI. If a true refractive index image is piecewise constant or piecewise polynomial in an ROI, the unique image reconstruction can be achieved via the TV or high-order TV minimization according to theorem 1.

### 3. Numerical simulations

#### 3.1. Forbild numerical phantom

The Forbild phantom was employed to evaluate the proposed numerical scheme. It consisted of 40 disks, as shown in figure 1(a). Each disk was assigned a different constant refractive index to mimic biological tissues in the range of \([0.1 \times 10^{-5}, 0.6 \times 10^{-5}]\). An ROI centered at the center of the phantom was selected to contain 128 \( \times \) 128 pixels, which covered about 25\% of the global area, as shown in figure 2(a). We adopted parallel-beam geometry for the
Figure 1. Forbild numerical phantom. (a) The numerical phantom and ROI; (b) the sinogram of differential projection of the phantom.

Figure 2. Interior reconstruction of a Forbild numerical phantom. (a) The reconstructed image using the proposed compressive sensing (CS)-based interior tomography algorithm; (b), (c) the profiles along the vertical line at $x = 55$ pixel and horizontal line at $y = 60$ pixel in the original phantom and the reconstructed image, respectively.

grating interferometric imaging mode, and equi-angularly acquired 361 projections over a 180° range. The detector array included 367 elements to collect the x-ray phase shift data. The projection of refractive indices can be numerically computed through equation (1). The differential phase shift data were computed using the difference method from the projection
data of refractive indices and corrupted by Gaussian noise to yield the truncated differential phase shift data with a signal-to-noise ratio of 15 dB, as shown in figure 1(b). Although the phase shift can be recovered from the differential phase shift data using integration methods, strong streak noises may be generated in the sinogram image, which would lead to an inferior reconstructed image of refractive indices. Here a new method was proposed to reconstruct the refractive index distribution directly using differential phase shift data. Based on theorem 2, the interior reconstruction can be formulated as the following optimization problem with the minimization of TV in the anisotropic norm (Cong et al. 2012):

$$\min \sum_{(i,j) \in \text{ROI}} (|\nabla_x \delta(i, j)| + |\nabla_y \delta(i, j)|)$$

subject to $\|\mathcal{F}^{-1}(2\pi iw \mathcal{F}(A_{\phi}))\delta - D_{\phi}\|_2 \leq \epsilon$, (17)

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ are the discrete forward and inverse Fourier transforms, respectively, $\mathcal{F}(A_{\phi})$ is a matrix that consists of the discrete Fourier transform of column vectors of $A_{\phi}$, $D_{\phi}$ is the differential phase shift data of $\phi$, $\delta$ is the differential phase shift data of $\phi$, $\nabla_x \delta$ and $\nabla_y \delta$ are partial derivatives of $\delta$ with respect to $x$ and $y$, respectively, $\mathcal{F}^{-1}$ is the discrete inverse Fourier transform, $\| \cdot \|$ is the $L_2$ norm, and $\epsilon$ is a tolerance parameter.

The $l_1$-norm regularization is a good sparsity measure of a refractive index image, and the split Bregman iterative algorithm is efficient to solve the $l_1$-norm optimization (Goldstein and Osher 2009):

$$\begin{align}
\delta^{k+1} &= \arg\min_{\delta} \left\{ \frac{1}{2} \| \mathcal{F}^{-1}(2\pi iw \mathcal{F}(A_{\phi}))\delta - D_{\phi}\|^2 + \frac{\lambda}{2} \| \nabla_x \delta - u_x^k + v_x^k \|^2 + \frac{\alpha}{2} \| \nabla_y \delta - u_y^k + v_y^k \|^2 \right\} \\
\frac{u_x^{k+1}}{u} &= \arg\min_{u} \left\{ \| u \|^2 + \frac{\alpha}{2} \| u - \nabla_x (\delta^{k+1} - v_x^k) \|^2 \right\} \\
\frac{u_y^{k+1}}{u} &= \arg\min_{u} \left\{ \| u \|^2 + \frac{\alpha}{2} \| u - \nabla_y (\delta^{k+1} - v_y^k) \|^2 \right\} \\
v_x^{k+1} &= v_x^k + \nabla_x (\delta^{k+1} - u_x^{k+1}) \\
v_y^{k+1} &= v_y^k + \nabla_y (\delta^{k+1} - u_y^{k+1}),
\end{align}$$

(18)

where $\lambda$ and $\alpha$ are regularization parameters. In each iteration, the first sub-problem only involves a least-squares problem, and can be efficiently solved. The rest of the sub-problems are the $l_1$-norm minimization, and can be solved via soft thresholding. We performed 100 iterations for the iterative scheme (18). The reconstructed image was in good agreement with the truth inside the ROI, as shown in figure 2(a). Representative profiles are shown in figures 2(b) and (c). Comparatively, we also performed a local FBP reconstruction from the same truncated differential phase shift data. It was found that the structure of the reconstructed refractive index image was in good shape but with more noise, as shown in figure 3(a). The major problem with the local FBP reconstruction from noisy truncated differential phase shift data is the substantial shifts in pixel values, as shown in figure 3(b).

3.2. Numerical human chest phantom

Furthermore, we conducted a numerical experiment for a realistic phantom to demonstrate the feasibility of the proposed approach. The phantom was produced based on a human chest CT slice of 620 $\times$ 620 pixels, which was obtained on a GE Discovery CT750 HD scanner at Wake Forest University Health Sciences, as shown in figure 4(a). The CT image values were converted to a refractive index distribution with a proper scale factor. We used parallel-beam geometry for the grating interferometric imaging mode, and equi-angularly acquired 361 projections over a 180° range. An ROI centered at coordinates of (340, 300) in the phantom was selected to contain 256 $\times$ 256 pixels, which covered about 17% of the global image. The detector array included 620 elements to collect x-ray phase shift data. The truncated differential phase shift data were computed via ray-tracing and corrupted with Gaussian noise to yield a signal-to-noise ratio of 25 dB. Using the iterative scheme in equation (18), the
Figure 3. Interior reconstruction of a Forbild numerical phantom. (a) The reconstructed image using the local FBP method; (b) the profiles along the horizontal line $y = 60$ pixel in the original phantom and the reconstructed image.

Figure 4. Interior reconstruction of a human chest numerical phantom. (a) The original phantom and ROI; (b) the reconstructed image using the proposed CS-based interior tomography algorithm; (c) the profiles along the vertical middle line corresponding to $x = 128$ pixel in the original phantom and the reconstructed image.
refractive index ROI image was reconstructed from the truncated differential phase shift data. The reconstructed image was in excellent agreement with the truth inside the ROI, and the detailed features in the ROI were quantitatively accurate except near the edge of the ROI, as shown in figures 4(b) and (c).

4. Discussion and conclusion

Truncated differential phase shift data can be produced by interior x-ray Talbot interferometric imaging or interior diffraction enhanced imaging. Our proposed techniques may be widely applied for interior biological soft tissue imaging, nondestructive test, food inspection, archaeometry and security screening. This interior imaging approach allows a substantial reduction of both radiation dose and system cost relative to global tomographic imaging.

Since it is difficult for human vision to recognize subtle fluctuations in an image, a piecewise constant (or polynomial) image model can often be an excellent approximation to biological tissues. In this sense, although biological tissues are highly heterogeneous, the refractive index image of biological tissues can be well reconstructed using the proposed method.

In x-ray propagation-based phase imaging, based on the transport intensity equation, measured intensity data can be used to compute the Laplacian of the phase shifts if an object has slowly varying phase and weakly absorbing properties (Wilkins et al 1996). Using the Laplace differential of the projections, the uniqueness and stability of x-ray refractive index interior tomography is an interesting future topic (Yang et al 2012).

In summary, we have demonstrated that the differential phase-contrast interior problem has a unique solution within the class of piecewise polynomial functions. In a numerical analysis framework, we have shown that an accurate interior reconstruction can be achieved via the TV (or high-order TV) minimization subject to the constraints of truncated differential projection data through an ROI. This approach allows the definition of TV in either anisotropic or isotropic form, offering flexibility for optimization. We have also developed a practical iterative algorithm for differential phase-contrast interior tomography. Our numerical results have established the feasibility of the proposed approach.

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