Can interior tomography outperform lambda tomography?

Whereas classic computed tomography (CT) theory targets the exact reconstruction of a whole cross-section or entire volume from complete projections, a real-world application often focuses on a region of interest (ROI). It has been a long-standing challenge to reconstruct an internal ROI only from truncated projections collected with a radiative beam through the ROI because this “interior problem” does not have a unique solution (1). When a traditional CT algorithm such as “filtered back-projection” is applied for an interior reconstruction from truncated projections, features outside the ROI may create artifacts overlapping inside features, rendering the images inaccurate or useless. On the other hand, over past decades, lambda tomography has been developed as a branch of applied mathematics that recovers gradient-like features within an ROI from truncated projections. With lambda tomography, the outcomes are not always the most appealing because of their non-quantitative nature. Recently, Quinto et al. (2) demonstrated the utility and limitation of electron lambda tomography and pointed out that “unless prior knowledge is being used...structures in the specimen cannot be exactly recovered even if we have access to noise-free continuum data...”

In 2007, our group (3) proved that the interior problem can be exactly and stably solved if a subregion in an ROI is known, which led to “interior tomography” for accurate ROI reconstruction in applications for which lambda tomography is intended. Similar results were also independently reported by others (4). However, precise prior knowledge of a subregion in an ROI can be unavailable in many cases. Hence, we have worked to relax the requirement of prior knowledge. In this direction, we were much inspired by the emerging compressive sensing theory to capture a compressible ROI image from truncated projections. Recently, we have analytically and experimentally shown that the interior problem indeed permits a unique solution if the ROI is piecewise constant or piecewise polynomial (5). Further extension along this line seems possible, such as with ROIs’ wavelet compressibility. We suggest that the piecewise polynomial assumption is already a rather generic image model (instead of a specific one based on precise knowledge of a subregion in an ROI), as the limited bandwidth model is for the current filtered backprojection CT methods. Therefore, it is our view that interior tomography would complement or outperform lambda tomography when quantitative information is desirable or critical because interior tomography is, in principle, exact if an ROI is angularly well irradiated, with or without precise subregion knowledge, or at least brings additional quantitative information relative to lambda tomography in the case of a limited angle scan. Our results are compared with those of Quinto et al. (2) in Fig. 1 and further illustrated in Fig. 2.

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Fig. 1. Limited angle interior reconstructions of the top (Left) and bottom (Right) examples in figure 2 of ref. 2, which were reconstructed from 120° local projection datasets via total variation minimization using the same scanning parameters as that for figure 2 of ref. 2.

Fig. 2. Further demonstration of limited angle interior tomography in a numerical test with a Shepp–Logan phantom (Upper) and an in vivo computed tomography study of sheep lungs (Lower) (also from 120° local projection datasets in the same way as that for Fig. 1), showing the filtered back projection reconstructions from complete projections (Left) and the counterparts from truncated data (Right) in good agreement over the region of interest. The sheep dataset is from Eric Hoffman, University of Iowa, Iowa City.
Reply to Wang and Yu: Both electron lambda tomography and interior tomography have their uses

Wang and Yu (1) suggest that interior tomography (IT) (2) could outperform electron tomography (ELT) (3) for electron tomography (ET) data. We suggest that IT could be useful for in vitro specimens (isolated macromolecular complexes in water) whereas ELT has been shown to be useful for in situ specimens (macromolecular complexes in complex cellular environments, as in tissue). We suggest that the tests reported in ref. 1 are not necessarily relevant to ET, although ET reconstructions of Rullgård (4) and Öktem suggest that IT might perform well on in vitro data.

(i) The main issues in ET are shot noise and clutter. Data are very noisy and the signal often has low-contrast against the background. The examples shown by Wang and Yu demonstrate that IT is effective on problems in which the signal has high contrast and data are low-noise or noise-free. Therefore, they do not demonstrate that IT outperforms ELT for ET, even though they demonstrate excellent performance for low-noise, high-contrast data. The good performance of ELT in ET originates mainly from ELT overemphasizing visible singularities and thereby increasing the contrast in the reconstructed signal (5).

(ii) We suggest that not all of the assumptions of IT are valid for ET. IT uses compressive sensing, which is based on the principle that a signal that is sparse with respect to some frame/basis is exactly recoverable with overwhelming probability from limited noise-free data whenever the measurement matrix fulfills the restricted isometry property (RIP). In limited angle tomography, the RIP does not hold, so the aforementioned principle does not apply to ET. Similarly, the uniqueness results in ref. 2 do not seem to apply to the limited-angle setting in ET. Furthermore, in ref. 2, uniqueness is proved by means of analytic continuation, an ill-posed procedure. IT gives exact reconstruction, with exact data, under assumptions that do not necessarily hold for ET data.

(iii) Our experience with TV regularization applied to ET, i.e., with IT, is that it can perform well on in vitro specimens by suppressing the background clutter (4). However, for in situ specimens, our experience suggests that the advantage of using IT is negligible compared with standard methods. The reason IT does not seem to be better for in situ specimens is probably that such specimens do not necessarily have a sparse gradient.

IT may be superior to ELT for in vitro specimens, but it needs to be tested on real ET data rather than on low-noise computed tomography (CT) data. ELT performs the best for in situ specimens where our experience shows that IT reconstructions can be hampered by severe staircasing (Fig. 1 iii). This problem might be overcome, but our experience suggests that would require modifying the TV function as in ref. 6. Most importantly, however, is that the signal-to-noise ratio is much smaller in ET data and that data, especially in situ ET data, are much more cluttered than the standard CT data on which IT is so effective.

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Fig. 1. Reconstructions from simulated data of an in vitro specimen containing tobacco mosaic virus (TMV). (i) A slice through the TMV phantom shown in iv. (ii) The best filtered backprojection reconstruction. (iii) The best IT reconstructions. It is clear that the central canal in the TMV is occluded in the IT reconstruction in iii due to staircasing. The simulated ET data are from H. Rullgård, Stockholm University.